# ANALYTIC STUDY OF THE TEMPERATURE CHANGE IN THE FLOWING MOLTEN METAL IN SAND CASTING

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A system of differential equations is given for the heat transfer in the flow of a liquid alloy in the channels in casting sand, and a formula is derived for the temperature of the alloy at any given point at an arbitrary instant.

There is considerable importance in the laws of heat transfer for flowing liquid metals, including metallurgy and casting. When a liquid alloy flows in a mold, the temperature changes, and this can affect the quality of the casting. We have examined analytically the temperature change during flow in rectangular channels in sand molds, which have low values for heat uptake.

To ensure that the mold is properly filled, the filling is often organized so that no crystallization occurs during the flow; therefore, we consider the heat transfer from a liquid superheated alloy (temperature above the liquidus point).

We consider a plane-parallel channel (Fig. 1). The mean-integral temperature over the cross-section is $t_1$, which is a function of the coordinate x and the time $\tau$, i.e., $t_1 = \varphi(x; \tau)$; to describe the heat transfer in the flow we have the equation

$$\frac{\partial t_1}{\partial x} w + \frac{\partial t_1}{\partial \tau} = -\frac{\alpha (t_1 - t_s)}{c_1 \rho_1 R}. \tag{1}$$

Here F/P = R, because the width of the channel considerably exceeds the thickness.

The heat transfer to a wall element occurs by thermal conduction; the thermal conductivity of the casting sand is very small, so longitudinal heat transfer in the mold wall is neglected, and so we use Fourier's equation

$$\frac{\partial t_2}{\partial \tau} = a_2 \frac{\partial^2 t_2}{\partial y^2}. \tag{2}$$

The boundary and initial conditions are as follows; Eqs. (1) and (2) are meaningful for $\tau \geq x/w$, and if

$$\tau = 0, \text{then } x = w\tau_{\text{sta}} \text{ and } t_2 = t_0, \tag{3}$$

since the flow has reached only a certain point at the given instant, and the temperature there still retains its initial value. If

$$x = 0 \text{ ( inlet ), then } t_1 = t_{\text{in}}. \tag{4}$$

The boundary conditions are of the form

$$\lambda_2 \left( \frac{\partial t_2}{\partial y} \right)_{y=R} = -\alpha (t_1 - t_s), \tag{5}$$

$$\left( \frac{\partial t_2}{\partial y} \right)_{y=\infty} = 0. \tag{6}$$

In writing (6) we have taken into account the following feature: in casting, the wall of the mold is thick, and the material is of low conductivity, so the wall may be considered as semiinfinite in thickness.

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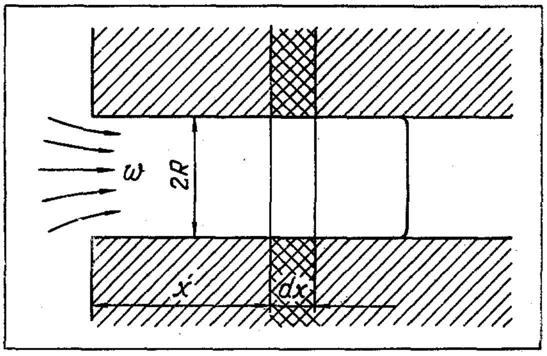


Fig. 1. The channel.

We introduce the new dimensionless variables

$$\xi = \frac{\alpha x}{c_1 \rho_1 w R}, \tag{7}$$

$$\eta = \frac{\alpha}{c_2 \rho_2 w R}(w\tau_{sta} - x) \tag{8}$$

and

$$r = y/R. \tag{9}$$

Then from (7) and (8) we get that

$$\frac{\partial t_1}{\partial x} = \frac{\partial t_1}{\partial \xi} \cdot \frac{\alpha}{c_1 \rho_1 w R} - \frac{\partial t_1}{\partial \eta} \cdot \frac{\alpha}{c_2 \rho_2 w R}, \tag{10}$$

$$\frac{\partial t_1}{\partial \tau} = \frac{\partial t_1}{\partial \eta} \cdot \frac{\alpha}{c_2 \rho_2 R}. \tag{11}$$

We substitute (10) and (11) into (1) to get after transformation that

$$\frac{\partial t_1}{\partial \xi} = -(t_1 - t_s). \tag{12}$$

Then (12) contains only the derivative $\partial t_1/\partial \xi$.

Equations (2)-(6) take the following form in the new variables:

$$\frac{\partial t_2}{\partial \eta} = \frac{\lambda_2}{\alpha R} \cdot \frac{\partial^2 t_2}{\partial r^2}, \tag{13}$$

$$\eta = 0, \; t_2 = t_0, \tag{14}$$

$$\xi = 0, \; t_1 = t_{in}, \tag{15}$$

$$\frac{\lambda_2}{R}\left(\frac{\partial t_2}{\partial r}\right)_{r=1} = -\alpha(t_1 - t_s), \tag{16}$$

$$\left(\frac{\partial t_2}{\partial r}\right)_{r=\infty} = 0. \tag{17}$$

The solution to (12)-(17) is sought by the method of integral Laplace transforms applied to each equation, and the result is written in the following form:

$$L[t_2(\xi;\; \eta;\; r)] = \int_0^\infty t_2 \exp(-s\eta)\, d\eta = F_2(s;\; \xi;\; r), \tag{18}$$

$$L[t_1(\xi;\; \eta)] = \int_0^\infty t_1 \exp(-s\eta)\, d\eta = F_1(s;\; \xi), \tag{19}$$

$$\frac{\partial F_1}{\partial \xi} = -(F_1 - F_n), \tag{20}$$

$$sF_2 - t_2(\eta = 0) = \frac{\lambda_2}{\alpha R} \cdot \frac{d^2 F_2}{dr^2}, \tag{21}$$

$$\eta = 0, \; F_2 = t_0/s,$$

$$\xi = 0, \; F_1 = t_{in}/s, \tag{22}$$

$$\frac{\lambda_2}{R}\left(\frac{\partial F_2}{\partial r}\right)_{r=1} = -\alpha(F_1 - F_n),$$

$$\left(\frac{\partial F_2}{\partial r}\right)_{r=\infty} = 0. \tag{23}$$

The solution to (19) with (14) and (23) is

$$F_2 = \frac{t_0}{s} + A_2 \exp\left[-\sqrt{\frac{\alpha R s}{\lambda_2}}\, r\right]. \tag{24}$$

At the metal–mold interface ($r = 1$), we have from (24) that

$$F_{2n} = \frac{t_0}{s} + A_2 \exp\left[-\sqrt{\frac{\alpha R s}{\lambda_2}}\right]. \tag{25}$$

We solve (22) with (24) and (25) for $F_1$ to get

$$F_1 = \frac{t_0}{s} + A_2 \exp\left(-\sqrt{\frac{\alpha R s}{\lambda_2}}\right)\left[\sqrt{\frac{\lambda_2 s}{\alpha R}} + 1\right]. \tag{26}$$

Also, in (24)-(26), $A_2 = \varphi(s, \xi)$.

We differentiate (26) with respect to $\xi$:

$$\frac{dF_1}{d\xi} = \frac{dA_2}{d\xi}\left[\sqrt{\frac{\lambda_2 s}{\alpha R}} + 1\right]\exp\left(-\sqrt{\frac{\alpha R s}{\lambda_2}}\right). \tag{27}$$

We solve (18) with (25) and (27) for $A_2$ to get

$$\ln A_2 = -\frac{\sqrt{\frac{\lambda_2 s}{\alpha R}}}{1 + \sqrt{\frac{\lambda_2 s}{\alpha R}}}\,\xi + C. \tag{28}$$

We get the expression for the arbitrary constant C from (28) with $\xi = 0$. We get the value for $A_2(0)$ from (26) with (21), and the result is

$$A_2 = \frac{(t_{\text{in}} - t_0)\exp\left[\frac{-\sqrt{\frac{\lambda_2 s}{\alpha R}}\,\xi}{1 + \sqrt{\lambda_2 s/\alpha R}}\right]}{s\exp\left(-\sqrt{\frac{\alpha R s}{\lambda_2}}\right)\left[1 + \sqrt{\frac{\lambda_2 s}{\alpha R}}\right]}. \tag{29}$$

We substitute (29) into (26) to get the following expression for the alloy temperature:

$$F_1 = \frac{t_0}{s} + \frac{(t_{\text{in}} - t_0)}{s}\exp\left[\frac{\sqrt{\frac{\lambda_2 s}{\alpha R}}\,\xi}{1 + \sqrt{\frac{\lambda_2 s}{\alpha R}}}\right]. \tag{30}$$

Equation (30) may be put in the form

$$F_1 = \frac{t_0}{s} + \frac{(t_{\text{in}} - t_0)}{s}\left[\exp(-\xi)\right]\exp\left(-\frac{\xi}{1 + \sqrt{\frac{\lambda_2 s}{\alpha R}}}\right). \tag{31}$$

Casting channels are usually short, so the combination $\xi = (\alpha x/c_1\rho_1 wR)$ is less than 1, and then to a high degree of accuracy we can replace $\exp(\xi/(1 + \sqrt{\lambda_2 s/\alpha R}))$ by $(1 + (\xi/(1 + \sqrt{\lambda_2 s/\alpha R})))$; the approximate expression for the alloy temperature in transform style is then

$$F_1 = \frac{t_0}{s} + \frac{(t_{\text{in}} - t_0)}{s}\left[1 + \frac{\xi}{1 + \sqrt{\frac{\lambda_2 s}{\alpha R}}}\right]\exp(-\xi). \tag{32}$$

The inverse transform is performed via tables [1], which give the following formula for the temperature of the moving alloy:

$$t_1 = t_0 + (t_{\text{in}} - t_0)\exp(-\xi)\left\{1 + \xi\left[1 - \exp\left(\frac{\alpha R \eta}{\lambda_2}\right)\operatorname{erfc}\left(\sqrt{\frac{\alpha R}{\lambda_2}\,\eta}\right)\right]\right\}. \tag{33}$$

Then the meanings of $\xi$ and $\eta$ are given by (7) and (8), which means that

$$t_1 = t_0 + (t_{\text{in}} - t_0)\exp\left(-\frac{\alpha x}{c_1\rho_1 wR}\right)\left\{1 + \frac{\alpha x}{c_1\rho_1 wR}\left[1 - \exp\left(\frac{\alpha^2(w\tau_{\text{sta}} - x)}{c_2\rho_2\lambda_2 w}\right)\operatorname{erfc}\sqrt{\frac{\alpha^2(w\tau_{\text{sta}} - x)}{c_2\rho_2\lambda_2 w}}\right]\right\}. \tag{34}$$

Equation (34) is strictly correct if the heat-transfer coefficient is constant; in fact, the heat transfer is of transient type, but experimental evidence on the transfer under such circumstances shows that the temperature at the interface rises almost instantaneously from the initial value to some value approaching that in the body of the alloy as the front advances, with only very slow subsequent change. At any given point in a channel, the alloy temperature and the surface temperature of the wall rise slowly as time passes (a few degrees a second), so the process is nearly steady-state, except for the initial instance near $\eta = 0$. To obtain correct results, one has to use experimentally measured values for the heat-transfer coefficient, which effectively takes into account the above features. We have [2] derived the following formula for the temperature at the end of the alloy flow:

$$t_1 = t_0 + (t_{in} - t_0)\exp\left(-\frac{\alpha_1 b_m \tau_{sta}}{(b_a + b_m) c_1 \rho_1 R}\right). \tag{35}$$

We get an analogous formula from (34) by putting $x = w\tau_{sta}$ (end of flow):

$$t_1 = t_0 + (t_{in} - t_0)\exp\left(-\frac{\alpha\tau_{sta}}{c_1 \rho_1 R}\right). \tag{36}$$

Equations (35) and (36) coincide if in (34) we put

$$\alpha = \frac{b_m}{b_m + b_a}\alpha_1. \tag{37}$$

To calculate $\alpha_1$ we have [3] the formula

$$\alpha_1 = \frac{k w^n \lambda_1}{a_1^n R^{(1-n)}}. \tag{38}$$

For aluminum alloys, $k = 0.033$, $n = 0.82$.

To use (34) we should employ a heat-transfer coefficient calculated from (37) and (38).

We can simplify (34) for practical calculations; the quantity $\sqrt{\alpha^2(w\tau_{sta}-x)/c_2\rho_2\lambda_2 w}$ is fairly large for an alloy flowing in a sand mold (substantially more than 10) if $\tau_{sta} \gg x/w$, and for large values of the argument, one can expand the function erfc as the following series [1]:

$$\operatorname{erfc} u = \frac{1}{\sqrt{\pi}}\exp(-u^2)\left[\frac{1}{u} - \frac{1}{2u^3} + \frac{1\cdot 3}{2^2 u^5} - \frac{1\cdot 3\cdot 5}{2^3 u^7} + \ldots\right]. \tag{39}$$

As $u = \sqrt{\alpha^2(w\tau_{sta}-x)/c_2\rho_2\lambda_2 w}$ is large, we take only the first term in the brackets in (39), and (34) then becomes

$$t_1 = t_0 + (t_{in} - t_0)\exp\left(-\frac{\alpha x}{c_1\rho_1 wR}\right)\left\{1 + \frac{\alpha x}{c_1\rho_1 wR}\left[1 - \frac{1}{\sqrt{\pi}}\sqrt{\frac{c_2\rho_2\lambda_2 w}{\alpha^2(w\tau_{sta} - x)}}\right]\right\}. \tag{40}$$

This equation is not correct for the end of the flow; here one should use (36). A study of (40) and the equation for $dt_1/dx$ shows that the alloy temperature decreases as x increases but increases with $\tau_{sta}$, which corresponds to the physical essence of the phenomenon. The following is a practical calculation. The mold cavity enters through an inlet channel, and the temperature at the exit from that channel increases continuously; if that temperature has risen considerably by the end of the casting, then the part of the casting adjoining the inlet channel will show shrinkage defects (porosity). For practical purposes it is desirable to know the temperature at that point at the end of casting, and also have proper methods of reducing it. Consider the temperature at the exit from the casting channel into the mold for AL4 aluminum alloy. The initial data are: $t_{in} = 923$°K, $R = 0.005$ m, $x = 0.3$ m, $c_1 = 1050$ J/kg·deg, $\rho_1 = 2350$ kg/m$^3$, $\lambda_1 = 62.8$ W/m·deg, $a_1 = 2.5\cdot 10^{-5}$ m$^2$/sec, $w = 1.1$ m/sec, $\tau_{sta} = 30$ sec, $b_m = 1214$ W·sec$^{1/2}$/m$^2$·deg, $b_a = 12{,}448$ W·sec$^{1/2}$/m$^2$·deg.

From (37) and (38) we get $\alpha = 2977.6$ W/m$^2$·deg; we substitute the values into (40) to get that the temperature at the end of casting has risen at the end of the inlet channel to 989°K; if in (36) we replace $\tau_{sta}$ by $x/w = 0.3/1.1 = 0.27$, we get the temperature in a given section at the moment of entry of the alloy, which is 949°K, so the rise during casting is 40°, which is hazardous for the above reasons. We see from (40) that the temperature rise can be reduced by increasing the alloy flow speed, reducing the pouring time, or distributing the alloy to the mold through several inlets.

## NOTATION

$c_1$ is the specific heat of liquid alloy, J/kg · deg;
$\rho_1$ is the density of alloy, $kg/m^3$;
$\alpha$ is the heat transfer coefficient, $W/m^2 \cdot deg$;
$\alpha_1$ is the effective heat transfer coefficient, $W/m^2 \cdot deg$;
P is the channel cross-section circumference, m;
F is the cross-section of channel;
$t_s$ is the temperature at inner channel surface, °K;
w is the flow velocity, m/sec;
R is the half-thickness of channel, m;
$t_2$ is the temperature of mold wall, °K;
$\tau_{sta}$ is the time from start of alloy flowing in channel, sec;
$\lambda_2$ is the thermal conductivity of mold material, W/m · deg;
$t_0$ is the initial mold temperature, °K;
$c_2$ is the specific heat of mold material, J/kg · deg;
$\rho_2$ is the density of mold, $kg/m^3$;
$a_2$ is the thermal diffusivity, $m^2/sec$;
$b_m$, $b_a$ are the coefficients of heat accumulation by mold and alloy, $W/sec^{1/2}/m^2 \cdot deg$;
$t_{in}$ is the temperature of alloy at inlet, °K.

## LITERATURE CITED


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